Proceedings of the American Academy of Arts and Sciences.

Vol. 63. No. 10.—February, 1929.

THE EFFECT OF PRESSURE ON THE RIGIDITY OF STEEL AND SEVERAL VARIETIES OF GLASS.

By P. W. Bridgman.

THE EFFECT OF PRESSURE ON THE RIGIDITY OF STEEL AND SEVERAL VARIETIES OF GLASS.

By P. W. Bridgman.

Presented Oct. 10, 1928. Received Oct. 15, 1928.

CONTENTS.

Introduction.

The change of rigidity or shearing modulus under pressure seems to have never been determined. The measurement of this effect is not easy, and for a number of years I have been searching for a suitable method. With the development of the sliding contact potentiometer method of measuring small displacements,[1] which I have already applied to the measurement of various small motions under pressure, it was evident that a means was at hand, and in this paper the application of such a method is described. When I first approached the problem, I felt that a method which would give merely the order of magnitude of the effect would be worth while, and my attack was conducted in this spirit. It seemed probable that the effect of pressure on the shearing modulus of a very compressible substance such as glass would be much higher than on such an incompressible substance as steel, and that to a first approximation the effect on steel could be neglected in comparison with that on glass. The first method adopted was a differential method, therefore, which gave the difference between the effects on steel and glass; by setting the effect on steel equal to zero, I hoped to get a first approximation to the effect on glass. The numerical magnitudes obtained were not, however, in the expected range, so that it appeared very questionable whether the assumption of a vanishingly small effect on steel was justifiable. It became necessary, therefore, to devise a second method allowing an absolute deter-

mination on the steel which had been used as the standard of comparison. It turned out that the magnitude of the effect on glass does not differ by a very large amount from that on steel, so that there was no real necessity in having made the first measurements on a substance of so comparatively little physical interest as glass. I plan now to extend the measurements to other metals besides steel.

Experimental Methods.

In view of the smallness of the effect, it was necessary that the displacement, the change of which under pressure was to be measured, should be as large as possible. The helical spring satisfies this requirement, and also the necessary limitations imposed by the small size of the pressure apparatus, better than any other arrangement, such, for example, as the twist in a long slender rod. Furthermore, the extension of a helical spring of ordinary dimensions under load involves to a sufficient approximation only the geometrical dimensions and the shearing modulus, so that the required

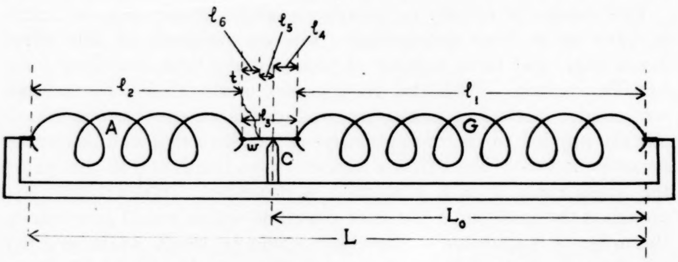

Figure 1. Scheme of the apparatus for determining the difference of the effect of pressure on the stiffness of two springs. If there is a change of relative stiffness under pressure, there is a shift of the point of coupling.

information could be calculated simply from the change of extension of the spring. The apparatus adopted for measuring the differential effect is shown in Figure 1. The glass spring G and the steel spring A are coupled together by a wire of manganin w. The springs are stretched to perhaps twice their normal length and attached at either end to a steel frame. The manganin wire slides over a contact C fixed to the steel frame, and there is a potentiometer terminal t attached to the wire. The entire assembly is exposed to hydrostatic pressure; if there is a change under pressure of the relative stiffness

of the springs, there will be a motion of the point of coupling. If A becomes relatively stiffer than G, A shortens while G is lengthened, and conversely. The amount of this motion is measured in the conventional way on a potentiometer, current passing lengthwise in w, and the difference of potential between C and t being determined.

The method for measuring the absolute effect, shown in Figure 2, is even simpler. The spring of steel is suspended vertically, carrying at the bottom end a weight P, separated from the spring by a length of manganin wire w, which carries the lead wire t and, as in the first method, slides on the contact C rigidly connected to the upper support of the spring. The whole arrangement is immersed in the liquid by which hydrostatic pressure is transmitted. If the shearing modulus increases or decreases under pressure the spring shortens or lengthens accordingly, and the amount of motion is obtained electrically from the difference of potential between C and t.

It is obvious that a number of experimental precautions must be observed to make such apparatus function properly, and there are a number of corrections to be applied in making the calculations. Perhaps the most obvious and important difficulty is friction at the sliding contact C. The changes are so slight, the whole motion under 12000 kg. being of the order of 0.5 mm., that very slight friction here would entirely mask the effect. To avoid error from friction, the differential apparatus was mounted horizontally in a pressure cylinder which could be rotated through 180° about the horizontal axis of the springs. The rotatable pressure apparatus which this demands had been already constructed, and was the same as that which had been used in measuring the viscosity of liquids; it could be used again for this purpose with only minor alterations. While the pressure was being changed, the cylinder was rotated through 180°, so that the contact C, Figure 1, was over the wire. In this position the weight of the springs and the wire produce a slight amount of sag, so that the contact was broken, and there is therefore no frictional resistance to the springs taking the exact position of equilibrium. To make the reading, the cylinder was rotated back through 180°, the wire w dropping back into contact with C, without encountering any friction. The contact so made was often so light that there were various coherer effects from outside disturbances, which made readings difficult. The resistance of the contact was decreased by passing through it current from a small bell-ringing magneto, in the same way that I have previously done in making resistance measurements.

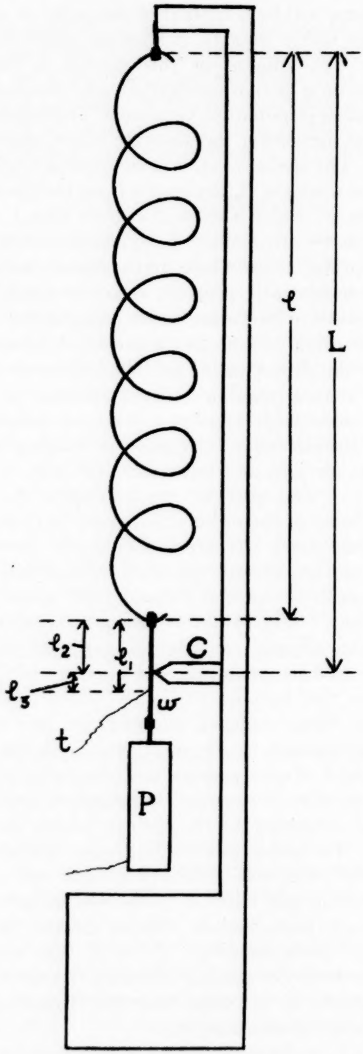

FIGURE 2. Scheme of the apparatus for determining the effect of pressure on the stiffness of a spring. The spring changes length when its stiffness changes.

In the case of the absolute apparatus (Figure 2), friction at the contact C during change of pressure was avoided by so mounting the apparatus that it could be very slightly tilted from the vertical, thus breaking contact.

It is evident that the various leads must be so flexible as to have negligible stiffness compared with the spring. The leads, t, were made of copper wire .004 cm. in diameter, wound into a long helix. The current connection at the glass spring end of the manganin wire in Figure 1 was a helix of this same wire, lying inside and concentric with the glass spring. The other current connection was made through the steel spring, which was grounded to the apparatus. In the absolute apparatus (Figure 2), the steel spring, grounded to the apparatus, was used for one current lead, and the other was a helix of the same fine wire grounded to the weight.

The measurements were made in a straight forward manner, and do not require detailed description. The pressure range was 12000 kg./cm.[2] The temperature of most of the measurements was 30°, which was maintained with a thermostat. Rough measurements were made on two of the specimens to find whether there was any large temperature effect. Measurements were usually made at 2000 kg. intervals in the order: 0, 4000, 8000, 12000, 10000, 6000, 2000, and 0. The accuracy was not sufficient to justify an attempt to determine departures of the effect from linearity.

Corrections and Method of Calculation.

The corrections play an important part, so that it will be necessary to describe them in considerable detail. Consider first the differential method.

The force exerted by each spring is proportional to its extension from its unstressed length, and the condition of equilibrium when the two springs are coupled together is that the force exerted by the two springs should be equal. These conditions give

$$F_1 = k_1(l_1 - l_1'), \quad F_2 = k_2(l_2 - l_2'),$$

with the equilibrium condition

$$k_1(l_1 - l_1') = k_2(l_2 - l_2').$$

Besides the quantities which are sufficiently explained in Figure 1, l_1' and l_2' are the lengths of the two springs under no extensive force. The condition of equilibrium holds both before and after the application of pressure, so that we also have

$$(l_1 - l_1')dk_1 + k_1(dl_1 - dl_1') = (l_2 - l_2')dk_2 + k_2(dl_2 - dl_2'),$$

where the differentials are the increments produced by a given increment of pressure. Call now the linear compressibility of the two springs χ_1 and χ_2, that of the manganin χ_3, and that of the steel frame χ_4 (we choose the subscript 2 to denote the steel spring, so that for this particular apparatus χ_2 and χ_4 are to a sufficient approximation equal to each other). Using the condition that $l_1 + l_2 + l_3 = L$, and also that $dl_1' = -l_1'\chi_1 dp_1$, etc., a first equation is obtained:

$$(l_1 - l_1')dk_1 + k_1(dl_1 + l_1'\chi_1 dp)$$
$$= (l_2 - l_2')dk_2 + k_2\{(l_2'\chi_2 + l_3\chi_3 - L\chi_4)dp - dl_1\}. \quad (1)$$

In this equation, l_1, l_1', l_2, and l_2' are known from geometrical measurements of the apparatus; χ_2, χ_3, and χ_4 are known by independent experiment; k_1 and k_2 must be found by measurements of the extension of the spring under known weights; dk_2 is supposed known from the auxiliary experiments on the absolute pressure effect by the second method, and dp is the arbitrary increment of pressure, so that the only unknowns are dl_1 and dk_1. It is our next task to calculate dl_1 in terms of the measured change of resistance between the points C and t, thus permitting a determination of dk_1, and thus eventually of $d\mu_1$, the change of shearing modulus.

To connect dl_1 with the measured change of resistance, we have the relations $l_1 + l_4 = L_0$, and the similar equation obtained by differentiating this with respect to pressure; $l_4 + l_5 + l_6 = l_3$, and the corresponding differentiated equation; and $l_5 = R_0/\rho_0$ and the corresponding equation at pressure dp, which is

$$l_5 + dl_5 = \frac{R_0 + \Delta R_0}{\rho_0(1 + \alpha dp)}.$$

Combining these equations gives finally:

$$dl_1 = \frac{\Delta R_0}{\rho_0} + dp\left\{\chi_3(l_3 - l_6) - \chi_4 L_0 - \alpha\frac{R_0 + \Delta R_0}{\rho_0}\right\}. \quad (2)$$

In this equation, α is the pressure coefficient of the resistance of the wire w per unit length. It is the pressure coefficient of resistance measured in the usual way with terminals rigidly attached to the wire, corrected by the linear compressibility.

Let us next examine how to calculate dk_2 from the measurements with the absolute apparatus. By writing the equations $l + l_2 = L$, $l_2 = l_1 - l_3$, $l_3 = R_0/\rho_0$, and the corresponding equations after pres-

sure has been applied (see Figure 2 for notation), we find for the change of length of the spring in terms of the change of resistance, etc.:

$$dl = -\chi p L + \chi_m l_1 dp + \Delta R_0/\rho_0 - \{(R_0 + \Delta R_0)\alpha dp\}/\rho_0.$$

χ is the linear compressibility of the frame, and χ_m of the manganin wire.

To connect with the change in shearing modulus, we have the formula for a helical spring given by Miller:[2]

$$\mu = \frac{2Ps^3 \cos^2\alpha}{\pi a^4 \phi_0^2 (l - l_1)}.$$

Here

μ = shearing modulus of spring.
a = radius of the wire of the spring.
P = total longitudinal pull.
l' = initial length along axis of helix.
l = length under load along axis of helix.
α = angle between spires of spring and horizontal.
s = total length of spring wire ($l = s \sin \alpha$).
ϕ_0 = total angular twist of unstretched spring, in radians.

Differentiate this equation logarithmically, obtaining:

$$\frac{d\mu}{\mu} = \frac{dP}{P} + 3\frac{ds}{s} - 4\frac{da}{a} + 2\frac{d(\cos\alpha)}{\cos\alpha} - 2\frac{d\phi_0}{\phi_0} - \frac{d(l - l')}{l - l'}.$$

We have the relation

$$\frac{ds}{s} = \frac{da}{a} = -\chi dp.$$

If the material of which the helix is composed is in a state of ease, as we assume is approximately the case, then $d\phi_0 = 0$, because the effect of a change of pressure is merely a change of linear dimensions without a change of angle.

Using the connection between l and s, gives

$$\frac{dl}{l} = \frac{ds}{s} + \frac{\cos\alpha}{\sin\alpha} d\alpha.$$

This enables us finally to write:

$$\frac{d\mu}{\mu} = \frac{dP}{P} + \chi dp - 2\tan^2\alpha\left(\frac{dl}{l} + \chi dp\right) - \frac{dl}{l - l'} - \frac{l'\chi dp}{l - l'}.$$

On the right hand side everything may be found. dP is the change in the stretching force on the spring exerted by the weight. To calculate it, the density of the weight must be known and its compressibility, the density of the transmitting liquid, and its change under pressure.

Next to connect the change of shearing modulus, $d\mu$, with the change of stiffness of the spring, we find from the formula for μ that

$$k = \frac{\pi}{2}\,\frac{a^4 \mu \phi_0{}^2}{s^3 \cos^2 \alpha}.$$

Differentiate this logarithmically, using the various relations already employed, and we get

$$\frac{dk}{k} = -\,\chi dp(1 - 2\tan^2 \alpha) + 2\tan^2 \alpha \frac{dl}{l'} + \frac{d\mu}{\mu}. \qquad (3)$$

There are two relations of this kind, with the appropriate subscripts, one for the glass spring and one for the steel spring. For a given increment of pressure, $d\mu/\mu$ is, of course, determined entirely by the material, and does not depend on the particular geometry of the individual. For the steel spring, $d\mu/\mu$ is supposed known from the measurements with the absolute apparatus, so that for any particular experiment on the differential effect between glass and steel, dk/k for the steel spring (that is, dk_2/k_2) may be found. We are now in a position to return to equation 1, in which everything is now known except dk_1 for the glass spring. Solving this equation for dk_1, we now go back to equation (3) and solve for $d\mu/\mu$ for the glass, the quantity finally desired.

The magnitude of some of the correction terms is as great as that of the uncorrected effect. Thus in formula (2) for dl_1, if we call the uncorrected dl, $\Delta R_0/\rho_0$, the corrected dl will in some cases, where the pressure effect is comparatively small, be found to be of the opposite sign and greater numerically than the uncorrected dl. The largest part of the correction in this case arises from the term $\chi_4 L_0 dp$, that is, the term for the change of dimensions under pressure of the frame which holds the springs. More usually, however, the corrected dl differed from the uncorrected dl by something of the order of 20%. In finding $d\mu/\mu$ for steel by the direct method, we may call the term $dl/(l-l')$ the uncorrected effect. The actual effect was a little more than one half the uncorrected effect. By far the largest part of the correction here arises from the change with pressure of the buoyancy of the weight. The difference between $d\mu/\mu$ and

dk/k for the steel spring was of the order of 10%. In most cases $d\mu/\mu$ for the glass spring differed from its dk/k by something of the order of 20%.

Detailed Description of Experiments.

The Steel. Different steel springs were made for the absolute measurements and for each of the differential measurements. Greatest sensitiveness demands that the steel spring have such a stiffness that its total extension when coupled against the glass is the same as that of the glass, and since the stiffness of the glass springs varies greatly, it was necessary to vary the steel springs also. The general order of magnitude of the dimensions of the steel springs was: outside diameter 0.75 cm., length 1.5 cm., with 30 turns. The springs were wound in a lathe over mandrels of varying diameters; they were all made from the same coil of hard drawn piano wire, 0.025 cm. in diameter. Presumably the steel had a carbon content of about 1.25%.

The Glass. I am much indebted to Dr. Littleton, of the Research Laboratory of the Corning Glass Works, for providing six different varieties of glass. The compositions, supplied to me by W. C. Taylor, the chief chemist of the Corning Glass Works, were approximately as follows:

A is a potash lead silicate of very high lead content.

B is the same as Pyrex. A typical composition for this is:
SiO_2 81.4; B_2O_2 11.5; Na_2O 4.0; Al_2O_3 2.1; CaO 0.2; MgO 0.3.

C is a soda potash lime silicate.

D is a soda zinc borosilicate.

E is a soda lead borosilicate, opacified with calcium and aluminum fluorides.

F is a soda lime silicate containing a small percentage of boric oxide.

The glass was furnished in the form of solid rods of circular section, from 6 to 8 mm. in diameter. For converting these rods into helical springs I am very much indeed indebted to Professor Harold Pender, of the University of Pennsylvania, who developed the apparatus by which this was done, and to Dr. Charles Weyl, also of the University of Pennsylvania, who kindly supervised the actual work during the absence of Professor Pender in Europe. The glass rods were first drawn down to a diameter of the general order of 0.025 cm. The machine by which this is done consists essentially of a device by which the rod is fed through a brass casting maintained

at the softening temperature of the glass, and is pulled out on the further side and wound up on a wheel (the rod is so flexible in small diameter that it can be wound on a wheel of large diameter without breaking), rotating at a definite speed with respect to the feeding speed. In this way a slender rod is produced whose section is a controllable fraction of the section of the original rod. The slender rod is now wound into a helix in another specially constructed machine, which consists of a core of carbon on which the rod is wound, the core being rotated inside a brass casting maintained at the proper temperature, and fed transversely as it rotates in order to give the helix the proper pitch. As furnished me, the helixes were open wound, with about 15 turns per cm. For my purpose it was better to convert these into closely wound helixes, which I did by slipping inside the helix a closely fitting core of aluminum, compressing the helix with a weight sliding on the core, and then slowly warming in an electric furnace to the softening temperature. By watching the heating, it is easy to stop the operation at such a point that the turns are closely in contact, but without sticking to each other. Finally, the ends of the helix were bent so as to give the proper means of attachment at the ends by very circumspect manipulation with a microscopic gas flame, issuing from a steel capillary such as is used in hypodermic needles.

Compressibility of the Glass. One of the corrections involves the linear compressibility of the glass. Since this correction may be important, and since the compressibility of glass varies greatly with the composition, it was necessary to make a direct determination of the compressibility of each variety of glass. This was done with the apparatus which I have described as "the lever apparatus for short specimens," and which has been used in measuring many other compressibilities.[3] The glass was cut from the original rod to a length of 2.7 cm., and the ends ground flat. In most cases the rod was of such a diameter that it could be used without further modification, but in one or two cases where the original diameter was too high, the diameter of these rods was reduced by grinding by the proper amount; I did this in order not to introduce the internal strains which might have been the result of drawing down to the proper size by heating. Although not immediately needed, the compressibility was determined at two temperatures, 30° and 75°; these measurements of the compressibility of glass have a certain interest for their own sake, and supplement determinations which I have already published for glass of other compositions.[4] The results are shown in Table I.

TABLE I.

COMPRESSIBILITY OF GLASS.

Designation of Glass	$\Delta V/V_0 = -(ap + bp^2)$, pressure in kg/cm²					
	30° a	30° b	Deviation*	75° a	75° b	Deviation*
A	30.54×10^{-7}	-24.3×10^{-12}	.09%	30.60×10^{-7}	-22.6×10^{-12}	.07%
B	30.12	+ 6.1	.10	29.72	+ 6.7	.11
C	24.69	− 22.0	.44	25.44	− 21.2	.42
D	25.97	+ 4.0	.10	26.30	+ .2	.08
E	27.78	+ 2.5	.10	27.71	+ 3.8	.10
F	23.29	− 6.1	.07	23.92	− 10.1	.18

* This column shows the average deviations of a single reading from the smooth curve in terms of percentage of the maximum effect (at 12000 kg), and is an indication of accuracy.

Direct Measurement of the Shearing Modulus of Steel under Pressure. The steel spring for this determination had 48 turns, an outside diameter of 7.5 mm., and a length, when extended by the weight in a position to begin measurements, of 6.35 cm. The constant k was determined by hanging varying weights on the spring and observing the extension with a cathetometer; its value was 2.3, force being measured in gms., and extension in cm.

The liquid by which pressure is transmitted must be a perfect insulator, and should be relatively incompressible; furthermore its compressibility must be already known, or it must be specially determined. Kerosene was chosen as most nearly satisfying the several conditions. There is, however, a disadvantage in the use of kerosene, in that it becomes so viscous at high pressures that readings could not be made at pressures higher than 6000 kg., the contact refusing to open and shut when the apparatus was tilted at higher pressures. This defect could have been avoided by the use of petroleum ether, but the compressibility of this is much higher, and therefore the correction for the changing buoyancy of the weight is larger. Furthermore, the compressibility was not known with sufficient accuracy, so that, all things considered, the advantage seemed to lie with kerosene. The compressibility of kerosene was taken from a previous determination.[5]

The weight was composed of several metals, but was mostly of gold (7.073 gm. of gold, with 1.617 gm. of brass in the form of a holder for the gold, and 0.440 gm. of steel, the half weight of the spring). Gold was used in order to reduce the correction for the change of buoyancy with pressure to a low value. The correction was about 1% at the maximum pressure of 6000 kg. It is evident that with so small a correction the demands on the accuracy of the compressibility of the kerosene are not high. In addition to the correction arising from the compressibility of the kerosene, there is a correction arising from the compressibility of the metals of the weight; this was so small as to be just on the verge of the perceptible.

In Figure 3 all the observations are shown, the readings of the potentiometer slide wire from which the extension of the spring was obtained being plotted against the setting of the slider on the bridge by which pressure was measured in terms of the change of resistance of the manganin gauge. It will be seen that the relation between pressure and change of deformation of the spring is linear within a small margin of error. The change of length of the spring calculated from these readings was a shortening of 0.77 mm. at the

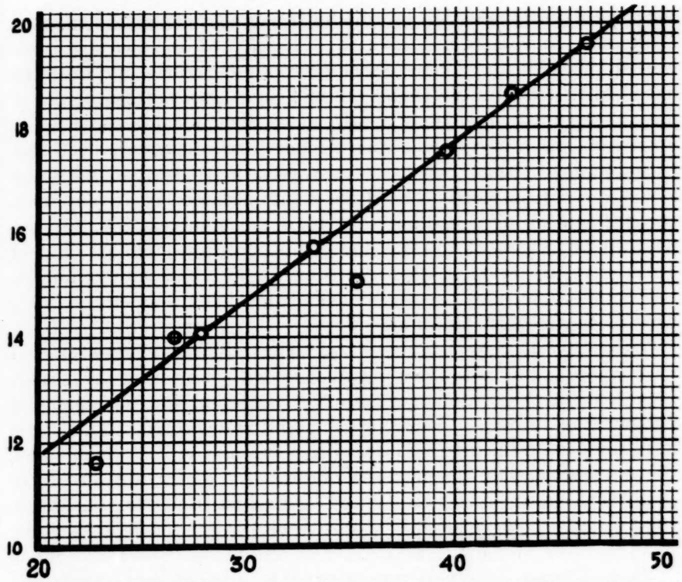

FIGURE 3. Reproduction of readings for the change of stiffness of a steel spring obtained with the apparatus of Figure 2. The ordinates are the settings in cm. on the potentiometer, and the abscissas the pressure in arbitrary units, the range of pressure between the extreme points being about 6000 kg. The extreme effect corresponds to an increase of shearing modulus of steel of about 1 per cent.

maximum pressure of 6000. From this, by means of the formulas already given, the change of shearing modulus is found to have the value:

$$\frac{1}{\mu}\left(\frac{\partial\mu}{\partial p}\right)_\tau = +\,2.16 \times 10^{-6},$$

pressure being expressed in kg./cm.² This means an increase of a little over 2% under a pressure of 10000 kg./cm.²

An attempt was made to obtain readings at 75° as well as at 30°. It was possible to run to higher pressures at 75°, because the viscosity of the kerosene is so much lower that there was no trouble from sticking contacts. Readings were made at 12000, 10000, and 8000, but at lower pressures nothing could be obtained because of

electrical disturbances due to the chattering contacts of the thermal regulator. At higher pressures the viscosity of the kerosene prevented this difficulty. The difficulty was not serious, but to have remedied it, troublesome changes would have been necessary, and it did not seem worth while, especially since the subject is to be taken up again, and the value of the temperature coefficient, if it had been obtained, could not have been used in connection with the measurements on glass. The three points obtained at 75° lay on a straight line of 40% smaller slope than the points at 30°, so that it is probable that at 75° the increase of shearing modulus of steel under pressure is materially less than at 30°, but one cannot be certain of this until all the corrections at the higher temperature have been more carefully determined.

Differential Measurements on Glass. The differential measurements were made, as has been already explained, with the axis of the springs in a horizontal position, so that the correction for the changing buoyancy of the transmitting liquid could be neglected. This made it possible to transmit pressure with petroleum ether, which offers the advantage that the increase of viscosity under pressure is so small that the measurements could be pushed to 12000. Readings were usually made only at 30°, but for two of the varieties of glass, readings were also made at 75°. The results for the various kinds of glass differed greatly in regularity; this is partly to be explained by differences in the absolute magnitude of the effect, and partly by the dimensions of the springs, the springs which were wound out of smaller diameter rods being obviously more sensitive to disturbing effects. A set of observations is reproduced in Figure 4; this is one of the better ones, although not the best.

In order to get as much idea as possible of the elastic properties of the various kinds of glass, the shearing modulus was calculated from the constants of the springs. The accuracy of this determination is low, and the results must be used only for orienting purposes. The chief source of error was in the diameter of the glass. This enters as the fourth power into the formula for the modulus, and could be determined only with low accuracy. The diameter of the actual glass of the spring could not be conveniently measured; instead I determined the mean diameter of several of the straight lengths left over from the winding operation, and these might vary by as much as 10% in one or two cases. As an additional check, the absolute shearing modulus of the steel was determined from the constants of the steel springs. There were six of these springs,

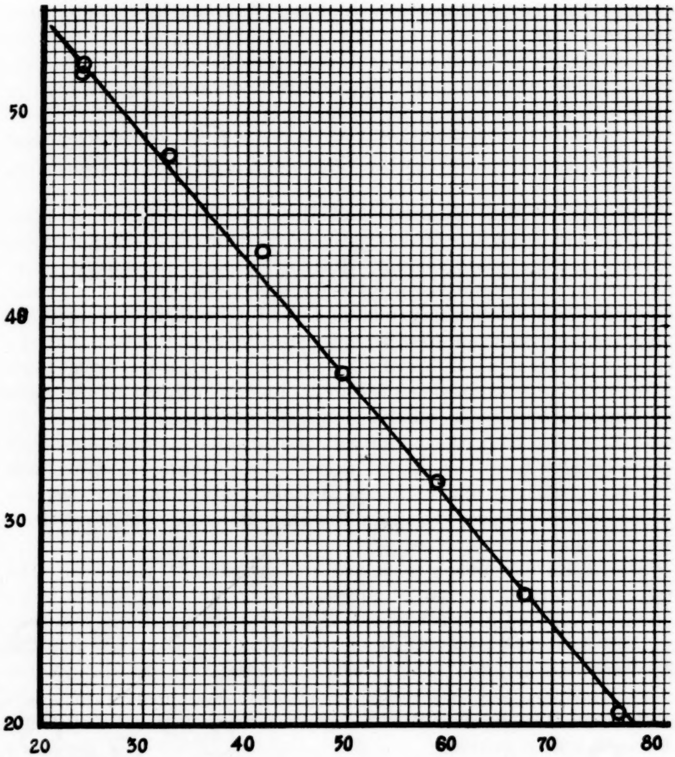

FIGURE 4. Reproduction of readings for the change of relative stiffness of steel and glass E, obtained with the apparatus of Figure 1. The ordinates are potentiometer settings in cm,, and the abscissas are pressures in arbitrary units, the range of pressure between the extreme points being about 12000 kg. The extreme effect corresponds to a decrease of rigidity of the glass with respect to the steel of about 10 per cent.

all of different dimensions, which were used to give the constant. The diameter of the steel wire may be safely assumed uniform since the wire was all from the same spool, but the chief source of error with the steel was in the outside diameter of the helix, which enters

as the third power into the modulus. The outside diameter was measured with a micrometer, but since the spring is very flexible, it was difficult to be sure that it was correctly obtained. The following values were found for the shearing modulus of the steel springs used with glass springs A, B, C, D, E, and F respectively: 6.77×10^{11}, 6.94, 7.99, 8.02, 7.30, and 7.45, average 7.4×10^{11}. Kaye and Laby's Tables gives for the shearing modulus of steel of 1% carbon content 8.12×10^{11}.

The final results obtained with the different varieties of glass are shown in Table II. The negative sign of the effect was a surprise to me; by very crude analogy with the action of pressure in enormously increasing the viscosity of liquids, I had expected a rather large increase of shearing modulus. On reflection, however, the negative sign does not seem so strange in view of the fact that the compressibility of a number of different kinds of glass has been shown to increase with increasing pressure.[4] In fact, turning to the table of compressibilities, it will be seen that just those glasses, B, D, and E, which have the abnormal increase of compressibility with pressure also have the largest decrease of shearing modulus, and the two glasses, A and C, which are most normal in their decrease of compressibility, also have the smallest numerical change of shearing modulus under pressure.

The temperature effect was measured on samples B and E. The only conclusion that can be drawn is that the effect is not large. B at 75° showed a displacement of the contact point 8% less than at 30°, and the displacement of the contact of E at 75° was 6% greater than at 30°. The effect of temperature on the various corrections was not determined, so that the statement above, that the temperature coefficient of the pressure coefficient is small, seems to be all that is justified.

Effect of Pressure on Other Elastic Constants.

Since an isotropic substance has only two independent elastic constants, we are now in a position to find the effect of pressure on Young's modulus, E, and Poisson's ratio, σ. We have the relations:

$$\sigma = \frac{3 - 2\mu c}{6 + 2\mu c},$$

$$E = \frac{9\mu}{3 + \mu c}.$$

TABLE II.

Designation of Glass	Shearing Modulus of Glass, in Abs. C. G. S. units	Pressure Coefficient of Shearing Modulus in kg. units, $\frac{1}{\mu}\left(\frac{\partial\mu}{\partial p}\right)_\tau$.	Probable Accuracy as Shown by Deviations of a Single Reading from a Smooth Curve	
			(a) in terms of percentage of maximum measured effect	(b) in terms of cms. displacement of contact point
A	2.60×10^{11}	-0.62×10^{-6}	15.2%	.0012 cm.
B	2.31	-8.45	.8	.0009
C	2.33	-2.15	4.8	.0017
D	3.14	-8.02	5.4	.0032
E	2.54	-8.80	1.4	.0010
F	(6.64)	-3.86	16.	.0021

Here μ is the shearing modulus, as before, and c is the volume compressibility, defined by the relation

$$c = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_\tau .$$

Differentiation of these equations gives:

$$\frac{1}{\sigma}\frac{d\sigma}{dp} = \frac{-9\mu c}{(3-2\mu c)(3+\mu c)}\left\{\frac{1}{c}\frac{dc}{dp}+\frac{1}{\mu}\frac{d\mu}{dp}\right\},$$

$$\frac{1}{E}\frac{dE}{dp} = \frac{1}{3+\mu c}\left\{3\frac{1}{\mu}\frac{d\mu}{dp} - \mu c\frac{1}{c}\frac{dc}{dp}\right\}.$$

$1/c\ dc/dp$ may be found from the formulas for compressibility, as in Table I, to have the value

$$a + \frac{2b}{a} \cdot$$

$\frac{1}{\mu}\frac{d\mu}{dp}$ is known, so that all the quantities are known which are required for the calculation of the pressure derivatives.

The velocity of a wave of shear is also of interest. This is given by

$$w = \sqrt{\frac{\mu}{\rho}},$$

whence:

$$\frac{1}{w}\frac{dw}{dp} = \frac{1}{2}\left[\frac{1}{\mu}\frac{d\mu}{dp} - c\right].$$

The results are contained in Table III. The calculation was not carried through for glass F because of the great uncertainty in the absolute value of its shearing modulus.

In general it appears that the changes in the various elastic constants are of the same order of magnitude as the change of compressibility already found. There is no general rule about the sign. The fact that the velocity of a wave of shear in glass decreases with increasing pressure may be of some geological interest.

I am indebted to my assistant, Mr. Stephen Stark, for most of the readings of this paper.

The Jefferson Physical Laboratory,
Harvard University, Cambridge, Mass.

TABLE III.

EFFECT OF PRESSURE ON VARIOUS ELASTIC CONSTANTS.

Substance	E Young's Modulus Abs. C. G. S.	σ Poisson's Ratio	$\frac{1}{E}\frac{dE}{dp}$ (p is in kg. /cm.^{2})	$\frac{1}{\sigma}\frac{d\sigma}{dp}$	$\frac{1}{w}\frac{dw}{dp}$ w is velocity of a wave of shear
Steel	20.8×10^{11}	.30	$+2.8 \times 10^{-6}$	$+2.6 \times 10^{-6}$	$+.78 \times 10^{-6}$
Glass A	6.2	.19	+2.2	+18.1	−1.8
" B	5.6	.22	−8.2	+1.5	−5.7
" C	5.9	.26	+0.7	+13.7	−2.3
" D	7.4	.18	−7.5	+3.3	−5.3
" E	6.2	.21	−8.0	+4.6	−5.8

REFERENCES.

[1] P. W. Bridgman, Proc. Amer. Acad. 58, 170, 175, 177, 1923.
Amer. Jour. Sci. 10, 483, 1925.

[2] J. W. Miller, Jr. Phys. Rev. 14, 146, 1902.

[3] Reference 1, page 175.

[4] P. W. Bridgman, Amer. Jour. Sci. 10, 359, 1925.

[5] P. W. Bridgman, Proc. Amer. Acad. 48, 357, 1912.